**Conversions**

1. **Rapid conversions**
2. **Substitution method**
3. **Successive divisions and multiplications**

1. **Rapid conversions:**conversions between bases which are powers of2.

* 1. **Conversion from the source base p=2k, p{4=22,8=23,16=24} into the destination base  2**

**Rule**:

**Each digit from the source number in base *p*=2*k*, the integer part and the fractional one, will be replaced by the corresponding group of *k* binary digits (adding if it is necessary insignificant zeros to the left).**

* 1. **. Conversion from base 2 into the destination base q=2k** , **q{4=22,8=23,16=24}**

**Rules:**

* **for the *integer/fractional  part*: from *right/left* to *left/right* (relative to the decimal point) make groups of *k*binary digits (eventually we add to the *left/right* insignificant zeros to have a complete group);**
* **the groups will be replaced by the corresponding digits in  base *q*=2*k.***

**Example 1:**

**(8) =      ? (2)       =  ? (16)**

**5327,321(8)=101 011 010 111,011 010 001(2)=**

**1010 1101 0111,0110 1000 1000(2)=AD7,688(16)**

**5(8)=5=4+1=2^2+2^0=        101 (2)**

**D(16)=13=2^3+2^2+2^0=1101(2)**

**115=64+32+16+2+1=2^6+2^5+2^4+2^1+2^0=1110011(2)**

**Example 2:**

**03 32 10,32 30  (4)  = 3E4,EC  (16)**

**32(4)=2 \* 4^0 + 3\*4^1 = 2 + 12 = 14 = E(16)**

**Example 3:**

**6BEA , BEC    (16) =  12 23 32 22, 23 32 30 (4)**

**B(16)=  11 = 2\*4^1 + 3 \* 4^0 = 23(4)**

**E(16)= 14 = 3\*4 + 2 = 32(4)**

1. **Substitution method**

* **calculations in the destination base**
* **it is recommended for *b < h*, *b* (source base),  *h*(destination base)**

***Steps:***

* **all the digits from the source representation are converted into the destination base:**
* **the base*b* is converted into base *h*:**
* **we calculate in base *h* the following sum:**

**Example 4:with a precision of 2 digits at the fractional part in the destination representation**

**1432,31(5)=        ,    (8)**

**1(5) = 1(8),  4(5) = 4(8), 3(5) = 3(8), 2(5) = 2(8), 5=5(8)**

**1432,31(5) = 1(8) \* 5(8) ^ 3 + 4(8) \* 5(8)^2 + 3(8) \* 5(8) ^ 1 + 2(8) \* 5(8) ^ 0 +**

**3(8) \* 5(8)^(-1) + 1 \* 5(8)^(-2)= 175 (8)+144(8)+17(8)+2(8)+ 0,46(8)+**

**+0,02(8) = 362,50(8)**

**!!! Calculations in base 8!!!**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **3** | **0** |  |  | **1** | **0** | **0** |  |  |  | 1 | 0 | **0** |  |  |  | **0** |  |  |  |  |  |  |  |  |  |  |  |
|  | **5** | **\*** |  |  | **3** | **1** | **\*** |  |  |  | **3** | **1** | **\*** |  |  | **3** | **\*** |  |  |  |  |  |  |  |  |  |  |
|  | **5** |  |  |  |  | **5** |  |  |  |  |  | **4** |  |  |  | **5** |  |  |  |  |  |  |  |  |  |  |  |
| **3** | **1** |  |  | **1** | **7** | **5** |  |  |  | **1** | **4** | **4** |  |  | **1** | **7** |  |  |  |  |  |  |  |  |  |  |  |

**5(8)^2=31(8)**

**5\*5=25, 25 mod 8 = 1, carry = 3**

**5(8)^3=  175(8)**

**3\*5 = 15, 15 mod 8 = 7, carry = 1**

**4\*3=12,12 mod 8 =4 , carry 1**

**3(8)\*5(8)=17(8)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **3,00 (8)** | **: 5(8)** | **1,00(8)** | **5(8)** | **0,14(8)** | **5(8)** |  |
| **/** | **0,46 (8)** | **/** | **0,14** | **/** | **0,02** |
| **30** | | **10** | | **1** | |
| **/** | | **/** | | **/** | |
| **40** | | **30** | | **14** | |
| **2** | | **4** | | **2** | |

           30(8)=3 \* 8 + 0 = 24, 24 div 5 = 4, 24 mod 5 = 4

40(8) = 4 \* 8 + 0 = 32, 32 div 5 = 6, 32 mod 5 = 2

**10(8) = 1 \* 8 = 8 = 8, 8 div 5 =1, 8 mod 5 = 3**

**30(8) = 3 \* 8 = 24, 24 div 5 = 4, 24 mod 5 = 4**

**14(8) = 8 + 4 = 12, 12 div 5 = 2, 12 mod 5 = 2**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **c** | **1** | **2** | **0** |  | **1** | **0** |  |
|  | **1** | **7** | **5** | **,** | **0** | **0** |  |
|  | **1** | **4** | **4** | **,** | **0** | **0** |  |
|  |  | **1** | **7** | **,** | **0** | **0** |  |
|  |  |  | **2** | **,** | **0** | **0** |  |
|  |  |  | **0** | **,** | **4** | **6** |  |
|  |  |  | **0** | **,** | **0** | **2** |  |
|  | **3** | **6** | **2** | **,** | **5** | **0** |  |

**0+5+4+7+2+0+0 = 18 (10)   18 div 8 = 2 , 18 mod 8 = 2**

**Example 5:with a precision of 2 digits at the fractional part in the destination representation**

**1562,34    (7) =    278,81(16)**

**1(7)=1(16), 5(7) …., 7=7(16)**

**1562,34(7)=1(16)\*7(16)^3+5(16)\*7(16)^2+6(16)\*7(16)^1+2(16)\*7(16)^0+3(16)\*7(16)^-1+4(16)\*7(16)^-2=157(16)+F5(16)+2A(16)+2(16)+0,6D(16) + 0,14(16) = 278,81(16)**

 !!! Calculations in base 16

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| c | 3 | 0 |  |  | 1 | 0 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 7 | \* |  |  | 3 | 1 | \* |  | 3 | 1 | \* |  | 7 | \* |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 7 |  |  |  |  | 7 |  |  |  | 5 |  |  | 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 3 | 1 |  |  | 1 | 5 | 7 |  |  | F | 5 |  | 2 | A |  |  |  |  |  |  |  |  |  |  |  |  |  |

7(16)^2= 31(16)

7\*7=49, 49 mod 16 =1,49 div 16 = 3

7(16)^3=31(16)\*7(16)=157(16)

7\*1=7

7\*3=21,m 21 mod 16 = 5, 21 div 16 = 1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 3,00 (16) | :7(16) | 4,00 | :7(16) | 0,92 | :7(16) |  |  |
|  | 0,6D |  | 0,92 |  | 0,14 |  |  |
| 30 | | 40 | | 09 | |  | |
| 60 | | 10 | | 22 | |  | |
|  | |  | |  | |  | |
|  | |  | |  | |  | |
|  | |  | |  | |  | |

60(16) = 6\*16 =96, 96 div 7 = 13(10) = D (16), 96 mod7 = 5

40(16) = 4\*16 = 64, 64 div 7 = 9, 64 mod 7 = 1

10(16) = 1\*16 + 0 = 16, 16 div 7 = 2, 16 mod 7 = 2

09(16) = 9(10), 9 div 7 = 1, 9 mod 7 = 2

22(16) = 2\*16 + 2 = 34, 34 div 7 = 4, 34 mod 7 = 6

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **carries** | 1 | 1 | 0 |  | 1 | 0 | +(16) |
|  | 1 | 5 | 7 | , | 0 | 0 |  |
|  |  | F | 5 | , | 0 | 0 |  |
|  |  | 2 | A | , | 0 | 0 |  |
|  |  |  | 2 | , | 0 | 0 |  |
|  |  |  | 0 | , | 6 | D |  |
|  |  |  | 0 | , | 1 | 4 |  |
|  | 2 | 7 | 8 | , | 8 | 1 |  |

  13+4 = 17(16)

1. **The method of successive divisions/multiplications**

* **it is recommended for *h < b*, *b* –source base and *h*- destination base.**
* **calculations in the source base**

**Integer part**: **successive divisions** by the destination base (***h***) are performed

* the process of successive divisions ends when 0 is obtained as quotient.
* the remainders, in the reverse order, are the digits of the new representation in base *h.*

**Fractional part**: **successive multiplications**by the destination base (***h***) are performed

* the fractional part is multiplied by ***b*** obtaining a number with an integer part and a fractional one;
* we continue with the multiplication of this new fractional part,...
* the process of the successive multiplications continues until one of the following conditions is satisfied:

          a)  the fractional part becomes 0;

          b) an established number of digits of the fractional part were calculated;

          c) periodicity is obtained.

* the integer parts, in the order of obtaining them during the multiplications process, are the digits of the fractional part in the destination representation.

**Example 6: with a precision of 3 digits at the fractional part in the destination representation**

**362,50(8)= ~ 1432,303 (5)**

 !! Calculations in base 8

**Conversion of the integer part**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **362(8)** | :5(8) | 60(8) | :5(8) |  | 11(8) | :5(8) | 1(8) | :5(8) |
| 36 | 060 | 10 | 11(8) |  | 11 | 01 | 1 | 0 |
| 02 | | 3 | |  | 4 | |  | |
| 2 | |  | |  |  | |  | |
|  | |  | |  |  | |  | |
|  | |  | |  |  | |  | |

**Calculations:**

 03(8)= 3

 36(8)=3\*8+6 = 24+6=30(10), 30/5=6, 30%5=0

10(8)=1\*8+0=8, 8/5=1, 8%5=3

11(8)=1\*8+1=9, 9/5=1, 9%5=4

**Conversion of the fractional part**  0,5(8)= 0, 303 (5)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **0,** | 5 |  | **\*** |  | 0, | 1 |  | **\*** |  |  |  |  | **\*** |  |
|  | 5 |  |  |  |  | 5 |  |  |  |  |  |  |  |  |
| 3, | 1 |  |  |  | 0, | 5 |  |  |  |  |  |  |  |  |

**Calculations:**

 0,5(8)\*5(8)=3,1(8)

 5\*5 = 25, 25/8 = 3, 25%8=1

 0,1(8) \* 5(8)= 0,5(8)0

**Example 7: with a precision of 3 digits at the fractional part in the destination representation**

**278  ,81(16)=~   1562, 33  (7)**

**Conversion of the integer part**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **278(16)** | :7(16) | 5A(16) | :7(16) | C(16) | :7(16) | 1(16) | :7(16) |
| 27 | 05A | 6 | 0C | 5 | 1 | 1 | 0 |
| 48 | |  | |  | |  | |
| 2 | |  | |  | |  | |
|  | |  | |  | |  | |
|  | |  | |  | |  | |

**Calculations:**

 27(16)=2\*16+7=39(10), 39/7=5, 39%7=4

48(16)= 4\*16+8= 64+8=72(10), 72/7=10=A(16), 72%7=2

5A()=5\*16+10=80+10=90, 90/7=12=C(16), 90%7=6

**Conversion of the fractional part**  0,81(16)=0,33 (7)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 0 |  |  |  | 3 | 0 |  |  |  |  |  |  |  |  |
| **0,** | 8 | 1 | **\*** |  | 0, | 8 | 7 | **\*** |  |  |  |  |  | **\*** |  |
|  |  | 7 |  |  |  |  | 7 |  |  |  |  |  |  |  |  |
| 3, | 8 | 7 |  |  | 3, | B | 1 |  |  |  |  |  |  |  |  |

**Calculations:**

 0,81(16)\*7(16)= 3,87(16)

8\*7=56(10), 56 div 16 = 3, 56 mod 16 = 8

0,87(16)\*7(16)=

7\*7=49, 49 div 16 = 3. 49 mod 16 = 1

8\*7=56, 56+3=59, 59 div 16 = 3, 59 mod 16 = 11